$1 + \sqrt{17}$ is not the Mahler measure of an algebraic number

Edward Dobrowolski [1]

The Mahler measure of a polynomial $f(x) = a \prod_{i=1}^{n} (x - \alpha_i) \in \mathbb{Z}[x]$ is defined as

$$M(f) = |a| \prod_{i=1}^{n} \max\{1, |\alpha_i|\},$$

so it is the product of the absolute value of the leading coefficient of f and the absolute values of the zeros of f lying outside of the unit circle.

The Mahler measure of an algebraic number α is define as the Mahler measure of its minimal polynomial. Mahler measure is generalized to polynomials in several variables via integral expression of the formula given above. In this case its values turned out to be related to the values of some $\mathfrak{L}$-functions and other interesting objects in number theory and currently is a vital area of research.

However, there are still lots of open problems about the values of Mahler measure of single variable polynomials and of algebraic numbers. Even the simplest case of quadratic irrational numbers is not fully understood. In 2004 A. Schinzel provided separately some necessary and some sufficient conditions for a quadratic algebraic number to be a Mahler measure, and wondered if his sufficient conditions are also necessary. He specifically asked about the number $1 + \sqrt{17}$ which falls in a gap between these conditions. In this talk we show that it is not the Mahler measure of an algebraic number.

[1] Department of Mathematics and Statistics, University of Northern British Columbia, Prince George, B.C., V2N 4Z9, Canada.